Name (IN CAPITALS): **Version #1**

Instructor: Dora The Explorer

Math 10550 Exam 3
Nov. 21, 2024.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off (and Put Away) all cellphones, smartwatches and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 18 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)
11.	(•)	(b)	(c)	(d)	(e)
12.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ________

13. ________

14. ________

15. ________

16. ________

Total ________

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1.	(a)	(b)	(c)	(d)	(e)
2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________________
13.	________________
14.	________________
15.	________________
16.	________________
Total	________________

Multiple Choice

1.(7pts) The inflection points of the curve $y = \frac{x^4}{2} - 5x^3 + 18x^2 + 6x + 6$ are at:

$y' = 2x^3 - 15x^2 + 36x + 6, y'' = 6x^2 - 30x + 36 = 6(x-2)(x-3)$. Since y'' changes sign at both $x = 2$ and $x = 3$, the inflection points are $x = 2$ and $x = 3$.

(a) $x = 2$ and $x = 3$

(b) $x = 1$ and $x = 2$

(c) $x = 1$, $x = 2$ and $x = 3$

(d) $x = 3$ only

(e) The graph has no inflection points.

2.(7pts) Compute $\lim_{x\to\infty} \frac{x+1}{\sqrt{x^2+x}+\sqrt{x^2-1}}$.

$$\lim_{x\to\infty} \frac{x+1}{\sqrt{x^2+x}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}}+\sqrt{1-\frac{1}{x^2}}} = \frac{1}{\sqrt{1}+\sqrt{1}} = \frac{1}{2}.$$

(a) $1/2$ (b) 1 (c) -1 (d) 0 (e) ∞

3.(7pts) The equation of the slant asymptote of the curve

$$y = \frac{2x^3 + x^2 - 2x}{x^2 - 1}$$

is:

$$\frac{2x^3 + x^2 - 2x}{x^2 - 1} = 2x + 1 + \frac{1}{x^2 - 1}$$

Hence, the slant asymptote of the curve is $y = 2x + 1$.

(a) $y = 2x + 1$ (b) $y = 2x$ (c) $y = x + 1$ (d) $y = 2x - 1$ (e) $y = x - 3$

4.(7pts) What is the minimum distance between the point $(4, 0)$ and the points (x, y) on the curve $y = \sqrt{2x}$?

The square of the distance function can be written as

$$D(x)^2 = (4 - x)^2 + \sqrt{2x}^2 = 16 - 8x + x^2 + 2x = x^2 - 6x + 16$$

Taking derivative we get

$$2D(x)D'(x) = 2x - 6.$$

Hence, the minimum distance is achieved when $x = 3$. We get $D(3) = \sqrt{7}$.

(a) $\sqrt{7}$ (b) $\sqrt{5}$ (c) $\sqrt{3}$ (d) 1 (e) 4

5.(7pts) Newton's method is used to find a root of the equation

$$x^3 - 2x - 3 = 0.$$

If $x_1 = 2$, find x_2.

$$x_2 = x_1 - \frac{x_1^3 - 2x_1 - 3}{3x_1^2 - 2} = \frac{19}{10}.$$

(a) 19/10 (b) 21/10 (c) 11 (d) 28/19 (e) 48/19

6.(7pts) Calculate the following indefinite integral $\int \frac{1+x}{\sqrt{x}} dx$.

$$\int \frac{1+x}{\sqrt{x}} dx = \int x^{-\frac{1}{2}} + x^{\frac{1}{2}} dx = 2x^{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{3} + C$$

(a) $2\sqrt{x} + \frac{2x^{3/2}}{3} + C$

(b) $\sqrt{x} + x^{3/2} + C$

(c) $\frac{\sqrt{x}}{2} + \frac{3x^{3/2}}{2} + C$

(d) $\frac{-1}{2x^{3/2}} + \frac{1}{\sqrt{x}} + C$

(e) $\frac{-1}{2x^{3/2}} - \frac{1}{\sqrt{x}} + C$

7.(7pts) Find the right endpoint approximation to the area under the curve

$$y = \frac{1}{1+2x^2}$$

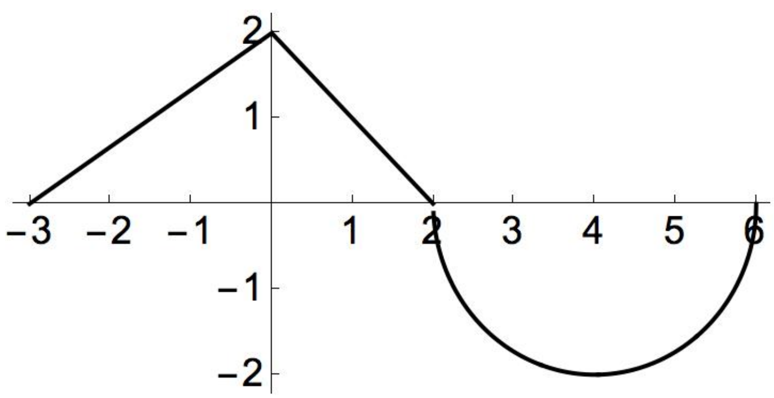
on the interval $-1 \le x \le 1/2$ using three approximating rectangles with equal base length.

The width of each of our rectangles is $\frac{\frac{1}{2}--1}{3} = \frac{\frac{3}{2}}{3} = \frac{1}{2}$. Since we are using the right endpoint approximation, we use the following x-values for our sum: $\frac{-1}{2}, 0, \frac{1}{2}$. Now we have the following sum:

$$\begin{aligned} R_3 &= \frac{1}{2} \cdot \Big(\frac{1}{1+2(\frac{-1}{2})^2} + \frac{1}{1+2(0)^2} + \frac{1}{1+2(\frac{1}{2})^2}\Big) \\ &= \frac{1}{2} \cdot \Big(\frac{1}{\frac{3}{2}} + 1 + \frac{1}{\frac{3}{2}}\Big) \\ &= \frac{1}{2} \cdot \frac{7}{3} = \frac{7}{6}. \end{aligned}$$

(a) 7/6 (b) 1 (c) 7/3 (d) 2 (e) 3/2

8.(7pts) The graph of $g(x)$ shown below consists of two straight lines and a semicircle. Use it to calculate the integral $\displaystyle\int_{-3}^{6} g(x)dx$.



The value of $\int_{-3}^{6} g(x)dx$ can be found by directly computing the area of the region between $g(x)$ and the $x-$axis on the interval [-3, 6]. We break the region up into two right triangles and a semi-circle. The area of the left triangle is $\frac{1}{2} \cdot 3 \cdot 2 = 3$. The area of the right triangle is $\frac{1}{2} \cdot 2 \cdot 2 = 2$, and the area of the semicircle is $\frac{\pi \cdot 2^2}{2} = 2\pi$. Since both triangles are above the x-axis, their areas remain positve. However, we negate the area of the semi-circle since it lies below the x-axis. Thus, we have $\int_{-3}^{6} g(x)dx = 3 + 2 - 2\pi = 5 - 2\pi$.

(a) $5 - 2\pi$ (b) $5 + 2\pi$ (c) $4 - 4\pi$ (d) $4 + 4\pi$ (e) 3

9.(7pts) If $\int_{-1}^{1} f(x)\,dx = 2$, $\int_{1}^{5} f(x)\,dx = 1$ and $\int_{-1}^{0} f(x)\,dx = -3$, find

$$\int_{0}^{5} f(x)\,dx.$$

By the additivity property of definite integrals, we know

$$\int_{-1}^{1} f(x)dx + \int_{1}^{5} f(x)dx = \int_{-1}^{5} f(x)dx.$$

So, $\int_{-1}^{5} f(x)dx = 2+1 = 3$. Similarly, we have $\int_{-1}^{5} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{5} f(x)dx$. We can now solve for $\int_{0}^{5} f(x)dx$. Therefore, $\int_{0}^{5} f(x)dx = \int_{-1}^{5} f(x)dx - \int_{-1}^{0} f(x)dx = 3-(-3) = 6$.

(a) 6 (b) 0 (c) 3 (d) -1 (e) -2

10.(7pts) Let $f(x) = \int_{0}^{x^2} \sqrt{1+\cos(t)}\,dt$. Find $f'(x)$.

We use chain rule since there is a composition of functions here. Our outside function is $f(x)$, and our inside function is x^2. Combining this with the fundamental theorem of calculus we have $f'(x) = \sqrt{1+\cos(x^2)} \cdot 2x$.

(a) $2x\sqrt{1+\cos(x^2)}$

(b) $2x\sqrt{1+\cos(x)}$

(c) $\sqrt{1+\cos(x)}$

(d) $\sqrt{1+\cos(x^2)} - \sqrt{2}$

(e) $\sqrt{1-\sin(x^2)}$

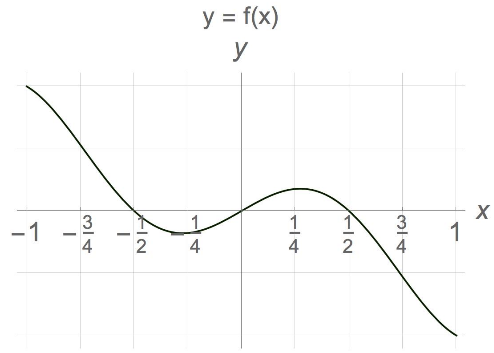
11.(7pts) Evaluate $\displaystyle\int_0^{\pi/3} x + 2\sin(x)\ dx$.

We have $\int_0^{\pi/3} x + 2\sin(x)\ dx = \frac{x^2}{2} - 2\cos(x)\Big|_0^{\frac{\pi}{3}} = \left(\frac{(\frac{\pi}{3})^2}{2} - 2\cos(\frac{\pi}{3})\right) - (0 - 2\cos(0)) = (\frac{\pi^2}{18} - 1) - (-2) = \frac{\pi^2}{18} + 1$.

(a) $\dfrac{\pi^2}{18} + 1$ (b) $\dfrac{\pi^2}{18} - 3$ (c) $\dfrac{\pi^2}{18} - \sqrt{3} + 2$

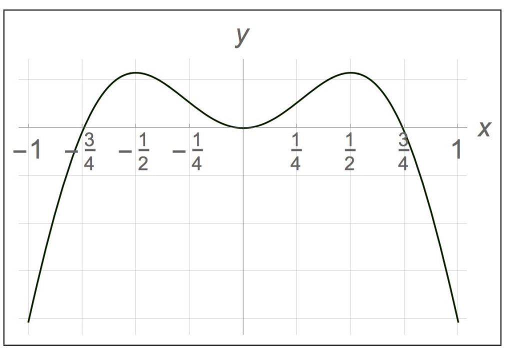
(d) $\dfrac{\pi^2}{18} - \sqrt{2}$ (e) $\dfrac{\pi^2}{3} - 1$

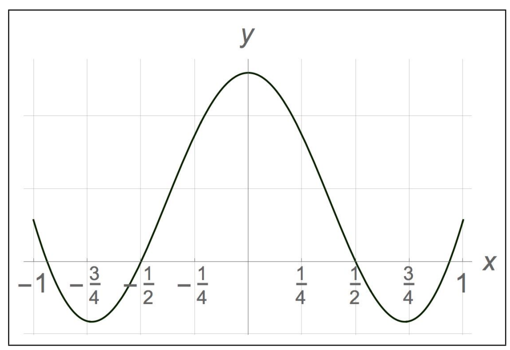
12.(7pts) The graph of $f(x)$ is shown below:

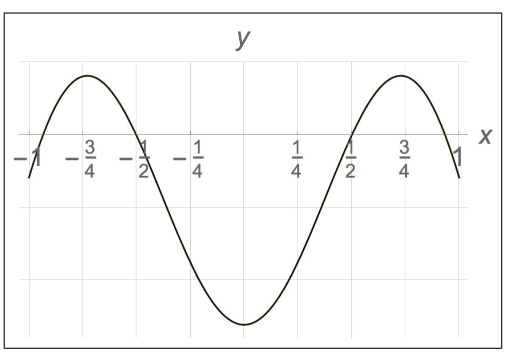


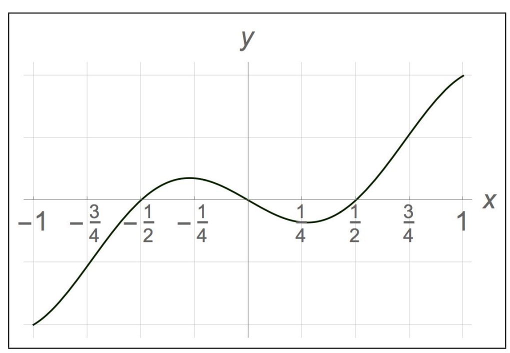
Which of the following gives the graph of an antiderivative for the function $f(x)$?

Recall that the antiderivative of $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$. Therefore, we can interpret the graph of $f(x)$ as the graph of the derivative of our desired function, and we want to find the graph of $F(x)$. First, we observe that $f(x) = 0$ at $x = \frac{-1}{2}, 0$, and $\frac{1}{2}$. So, $F(x)$ must have slope equal to 0 at exactly $x = \frac{-1}{2}, 0$, and $\frac{1}{2}$. This eliminates three graphs which have slopes equal to 0 at different x-values. Furthermore, we observe that $f(x)$ lies above the x-axis on the interval $[-1, \frac{-1}{2}) \cup (0, \frac{1}{2})$, so the slope of $F(x)$ must have a positive slope throughout these intervals. Of the two remaining graphs, there is exactly one which satisfies this requirement.

(a) 

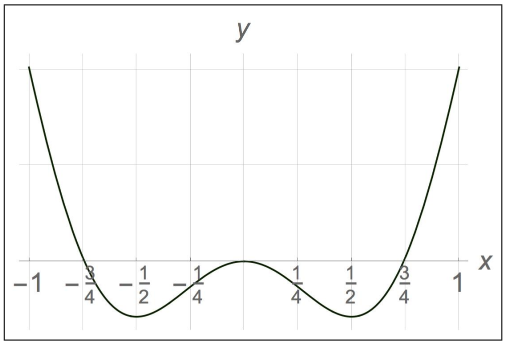
(b) 

(c) 

(d) 

9. Initials: ____________

(e)



Partial Credit

For full credit on partial credit problems, make sure you justify your answers.

13. (10pts) A ball is thrown straight upwards on the planet Zing from a height of 12 feet, at a velocity of 10 feet per second. The acceleration due to gravity on Zing is 4 feet per second per second, that is the acceleration of the ball is $a(t) = -4$ ft$/s^2$.

(a) Find the velocity function, $v(t)$, of the ball while it is in motion.

$v(t) = -4t+C$ for some constant C. Since $v(0) = 10$, then $C = 10$. Hence, $v(t) = -4t+10$.

(b) Find the height/position function, $h(t)$, of the ball while it is in motion.

$h(t) = -2t^2 + 10t + C$ for some constant C. Since $h(0) = 12$, then $C = 12$. Hence, $h(t) = -2t^2 + 10t + 12$.

(c) When does the ball hit the surface of the planet?

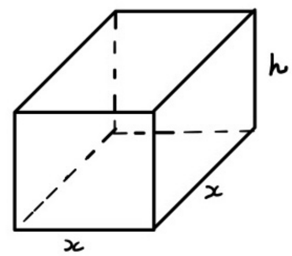
The problem is asking about the unique positive real root of the function $h(t)$. Notice

$$h(t) = -2t^2 + 10t + 12 = -2(t-6)(t+1).$$

Hence, the ball hits the surface at $t = 6$.

11. Initials: ____________

14.(12pts) If 300 cm^2 of material is available to make a box with a square base and an open top. Find the dimensions of the box with the largest possible volume (in cm^3).



We are given that the surface area of the box $SA = 300$. The surface area of our box is given by $SA = (\text{area of square base}) + 4 \cdot (\text{area of rectangular sides}) = x^2 + 4xh$. So, we have

$$\begin{aligned} x^2 + 4xh &= 300 \\ h &= \frac{300 - x^2}{4x} \\ h &= \frac{75}{x} - \frac{x}{4}. \end{aligned}$$

Since we would like to maximize the volume of our box, we need to find the critical points of our volume function. We have $V = x^2 h = x^2 \cdot \left(\frac{75}{x} - \frac{x}{4}\right) = 75x - \frac{x^3}{4}$. So, $V' = 75 - \frac{3x^2}{4}$, so $V' = 0$ when $x = \pm\sqrt{100} = \pm 10$. Since the domain of the function is $[0, \infty)$, the only candidate for the critical point is $x = 10$. Now we need to check if the derivative $V'(x)$ changes sign at $x = 10$.(One can also check values. For instance, $V'(0) = 75 > 0$ and $V'(12) = -33 < 0$.) Notice that $V''(10) < 0$, this implies that $x = 10$ is an absolute maximum. Plugging $x = 10$ back into our equation for h, we have $h = \frac{300 - 2(10)^2}{4 \cdot 10} = \frac{300 - 100}{40} = 5$.

13. Initials: ____________

14. Initials: ____________

True-False.

15.(6pts) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) If $v(t)$ gives the velocity of an object moving in a straight line, where $t > 0$, then $\int_0^2 v(t)dt$ always gives the distance travelled by the object over the time interval $0 \leq t \leq 2$.

False. Given the function $v(t) = -x + 1$. The distance traveled is 1 while

$$\int_0^2 v(t)dt = [-\frac{x^2}{2} + x]_0^2 = 0.$$

TRUE FALSE

(b)(1 pt. No Partial credit)

$$\int_0^{\frac{\pi}{2}} \sin(x)dx = 1$$

True.

$$\int_0^{\pi/2} \sin(x)dx = -\cos(x) \mid_0^{\pi/2} = 1$$

TRUE FALSE

(c)(1 pt. No Partial credit) If $f(x) = \int_1^x \frac{1}{t}dt,\ x > 0$, then $f'(x) = \frac{-1}{x^2}$.

False, by the fundamental theorem of calculus, we have

$$f'(x) = \frac{1}{x}.$$

TRUE FALSE

(d)(1 pt. No Partial credit) If $f(x) = \frac{(x-1)}{(x-2)}$, the the graph of $y = f(x)$ has vertical asymptotes $x = 1$ and $x = 2$.

TRUE FALSE

False. In rational functions, our vertical asymptotes will occur at the x-values where our function is undefined. We can observe from the denominator of $f(x)$ that $f(x)$ is undefined only at $x = 2$. Hence, $f(x)$ is defined at $x = 1$.

(e)(1 pt. No Partial credit) If $f(x) > 0$ and $f'(x) < 0$ on the interval $0 \leq x \leq 1$, then the left endpoint approximation (with 4 approximating rectangles) to the area beneath the graph of $y = f(x)$ on the interval $0 \leq x \leq 1$ is an overapproximation.

True. Sketch the graph of a decreasing function such as $f(x) = -x + 1$ to see this.

TRUE FALSE

(f)(1 pt. No Partial credit) Isaac Newton was the first person to discover that chocolate is a derivative of cocoa beans.

TRUE FALSE I'D RATHER NOT SAY!

16. Initials: ____________

16.(2pts) You will be awarded these two points if you write your name in CAPITALS on the front page and you mark your answers on the front page with an X through your answer choice like so: (a) (<u>not</u> an O around your answer choice) .

ROUGH WORK EXAM 3